Local Bifurcations in Ant Colonies

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Social Insects

Social insects display complex behavior: they build complex structures, scan their environments in search of available resources and perform group-based decisions (Deneubourg and Goss 1989).

As we will see, this self-organisation might originate from an exploitation of bifurcations and critical phenomena for internal organization (Deneubourg et al. 1989; Bonabeau 1996; Valverde et al. 2009).
Solving an Optimisation Problem for Food

\[ \begin{align*}
\frac{dx_1}{dt} &= \mu q_1 P_1(x_1, x_2) - \int x_2 \, dt \\
\frac{dx_2}{dt} &= \mu q_2 P_2(x_1, x_2) - \int x_1 \, dt
\end{align*} \]

where \( \mu \) is the rate of ants entering each branch from the nest, \( q_i \) the rate of pheromone deposition in the \( i \)-th branch, and \( \int \) the rate of pheromone evaporation.
Solving an Optimisation Problem for Food

Let \( x_1 \) and \( x_2 \) be the concentrations of pheromone in branch 1 and 2, respectively. Then (Nicolis and Denebourg 1999):

\[
\begin{align*}
\frac{dx_1}{dt} &= \mu q_1 P_1(x_1, x_2) - \nu x_1 \\
\frac{dx_2}{dt} &= \mu q_2 P_2(x_1, x_2) - \nu x_2
\end{align*}
\]

where \( \mu \) is the rate of ants entering each branch from the nest, \( q_i \) the rate of pheromone deposition in the \( i \)-th branch, and \( \nu \) the rate of pheromone evaporation.
Free Will

A possible response function is described by (Beckers et al. 1992):

$$P_i(x_1, x_2) = \frac{(x_i + K)^2}{\sum_{j=1,2}(x_j + K)^2}$$

(2)

$K$ is an empirically measured constant. Even the simpler symmetric pheromone deposition model ($q_1 = q_2 = q$) brings to some interesting insights.
Fixed Points

\[
\begin{align*}
\frac{dx_1}{dt} &= \mu q P_1(x_1, x_2) - \nu x_1, \\
\frac{dx_2}{dt} &= \mu q P_2(x_1, x_2) - \nu x_2
\end{align*}
\]  

(3)

There is a symmetric fixed point \( x^* \) when \( x_1 = x_2 \), with \( x^* = \frac{\mu q}{2\nu} \).

Other fixed points should be relative to the case \( x_1^* \neq x_2^* \), but still in agreement with

\[
\frac{x_1^*}{x_2^*} = \frac{P_1(x_1^*, x_2^*)}{P_2(x_1^*, x_2^*)} \rightarrow x_1^* (x_2^* + K)^2 = x_2^* (x_1^* + K)^2
\]  

(4)

By using \( x_1^* + x_2^* = \frac{\mu q}{\nu} \), then the other two fixed points are:

\[
x_1^* = \frac{\mu q}{2\nu} + \sqrt{\left(\frac{\mu q}{2\nu}\right)^2 - K^2}, \quad x_2^* = \frac{\mu q}{2\nu} - \sqrt{\left(\frac{\mu q}{2\nu}\right)^2 - K^2}
\]  

(5)
Pitchfork Bifurcation in Insect Behavior

\[ x_1^* = \frac{\mu q}{2\nu} + \sqrt{\left(\frac{\mu q}{2\nu}\right)^2 - K^2}, \quad x_2^* = \frac{\mu q}{2\nu} - \sqrt{\left(\frac{\mu q}{2\nu}\right)^2 - K^2} \]  

(6)

\( x_1^* \) and \( x_2^* \) exist if \( \frac{\mu q}{(2\nu)} > K \). Critical ant flow density value at:

\[ \mu_c = 2K\nu/q \]

Stability analysis reveals a *supercritical pitchfork bifurcation* for ant organisation.
Phase Transition

Fixing $K$ and $V$, the critical curve $\mu_c = \frac{2KV}{q}$ divides two phases, of ordered-disordered behavior in ants.

This is relative to a *symmetry breaking mechanism*, which was experimentally observed (Nicolis and Denebourg 1999).
No Synchronisation for Ants

Colonies of *Leptothorax* ants alternate periods of no movements to bursts of all-active individuals, in a periodic fashion.

Experiments indicate that the pattern to colony synchronisation is *not* present at the individual level, i.e. *ants do not act as interacting oscillators* (Strogatz 2003). A percolation model offers a more satisfying explanation.
Activation Spreading in a Mean Field case

A given ant can become active with rate $\alpha$, moving around and exciting neighboring ants. Active ants can become inactive with rate $\gamma$. Let $x$ be the fraction of active ants. Also, let $\rho$ be ant density.

Then (Solé et al., 1993):

$$\frac{dx}{dt} = \alpha x (\rho - x) - \gamma x.$$  \hspace{1cm} (7)

Two fixed points, namely $x_0^* = 0$ (no activity) and $x_1^* = \rho - \gamma / \alpha$.

Stability swap when:

$$\rho > \rho_c = \frac{\gamma}{\alpha}.$$  \hspace{1cm} (8)
Percolating Ants

A transcritical bifurcation happens, with *increased colony fluctuations* close to criticality, consistently with real data (Solé et al., 1993).
Conclusions

Mean-field simple ODE based models can describe some ant colony complex behavioral patterns in path routing and activation bursts.

PROS: analytical tractation of bifurcations/phase transitions; good agreement with experiments.

CONS: require well-mixed populations, missing stochastic fluctuations.
Thank you for your attention. Any questions?